An investigation into Invariance

In this activity you will explore how different vectors behave under transformations. Use Vectorama to observe patterns and describe what you notice.

- Open Vectorama (www.korovatron.co.uk/vectorama)
- Select **2D** and **POINT** mode

Exercise 1

- Add a 2D matrix that reflects in the x-axis.
- Add the following vectors: (2, -1), (2, 1), (2,3), (2,7)
- Apply the matrix a few times.

Describe what happens

Where does each point on the line x = 2 get mapped to under this transformation?
What might this line be called?

Exercise 2

- Change the vector display mode to **ARROW**
- Apply the same matrix used in exercise 1 to the same vectors.

Compare what you see now with what you observed in Exercise 1.

Exercise 3

- Switch to **3D** and keep the vector display mode set to **ARROW**
- Add the following vectors (1, 0, 0), (0, 1, 0), (0, 0, 1), (2, 0, 0), (0, 2, 0) and (0, 0, 2).
- Add the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ and apply it a couple of times.

Describe fully what happens to each vector, considering **direction** and **magnitude** and **orientation**.
Does the vector stay on the same line through the origin?

- Now delete the vectors, and replace with (2, 0, -2), (0, 4, 2) and (4, -2, -3).
- Apply the matrix.

Explain how these vectors behave differently from the first set.

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Exercise 4

- Switch to **2D** and keep the vector display mode set to **`ARROW`**
- Add the following vectors (1, 1), (-1, 2), (1, -2) and (-2, -2)
- Add the matrix $\begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$ and apply it a couple of times.

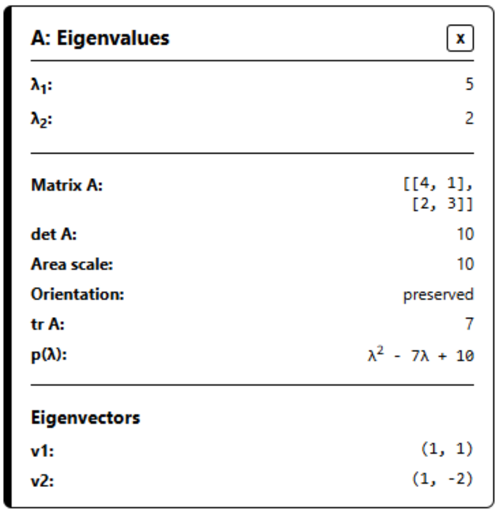
Describe fully what happens to each vector, considering **direction** and **magnitude** and **orientation**.

- Now delete the vectors, and replace with (2, 1), (0, 2) and (-3, -1)
- Apply the matrix.

Explain how these vectors behave differently from the first set.

Exercise 5

- Using the matrix from Exercise 3, click the small blue **`[i]`** button to open the matrix information panel.



In this panel you will see things called **`eigenvalues`** and **`eigenvectors`**.
Use what you observed in the previous exercises to explain what these quantities must represent.

- Open the information panel for the other matrices in this exercise, to confirm your answers.

Exercise 6

- At the bottom of the matrix information panel, you can display **`eigenspaces`** for the matrix.

How do the eigenspaces relate to the directions you observed earlier?