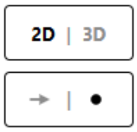
An investigation into Invariance

In this activity you will explore how different vectors behave under transformations. Use Vectorama to observe patterns and describe what you notice.

- Open Vectorama (www.korovatron.co.uk/vectorama)
- Select **2D** and **POINT** mode



Exercise 1

- Add a 2D matrix that reflects in the x-axis.
- Add the following vectors: (2, -1), (2, 1), (2,3), (2,7)
- Apply the matrix a few times.

Describe what happens

Where does each point on the line x = 2 get mapped to under this transformation? Another point on the same line
What might this line be called?
An invariant line

Exercise 2

- Change the vector display mode to **ARROW**
- Apply the same matrix used in exercise 1 to the same vectors.

Compare what you see now with what you observed in Exercise 1.
The direction of each vector changes. The purpose of this exercise is to emphasise that eigenvector => invariant but invariant does not imply eigenvector (common misconception)

Exercise 3

- Switch to **3D** and keep the vector display mode set to **ARROW**
- Add the following vectors (1, 0, 0), (0, 1, 0), (0, 0, 1), (2, 0, 0), (0, 2, 0) and (0, 0, 2).
- Add the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ and apply it a couple of times.

Describe fully what happens to each vector, considering **direction** and **magnitude** and **orientation**.
Does the vector stay on the same line through the origin? All preserve direction. Scales x3, x2, x1

- Now delete the vectors, and replace with (2, 0, -2), (0, 4, 2) and (4, -2, -3).
- Apply the matrix.

Explain how these vectors behave differently from the first set. Directions change

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Exercise 4

- Switch to **2D** and keep the vector display mode set to **`ARROW`**
- Add the following vectors (1, 1), (-1, 2), (1, -2) and (-2, -2)

- Add the matrix $\begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$ and apply it a couple of times.

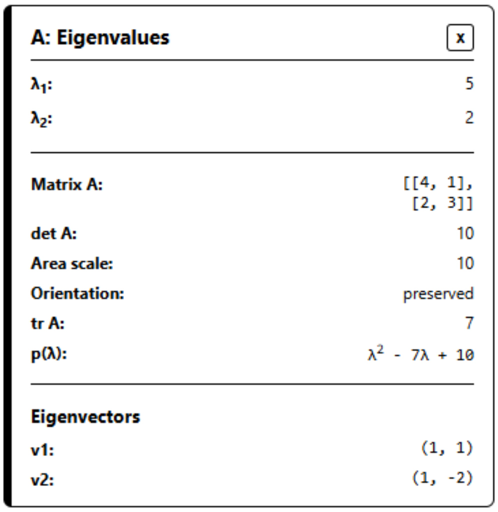
Describe fully what happens to each vector, considering **direction** and **magnitude** and **orientation**.
Preserves direction. Scales x5, x2

- Now delete the vectors, and replace with (2, 1), (0, 2) and (-3, -1)
- Apply the matrix.

Explain how these vectors behave differently from the first set. Directions change

Exercise 5

- Using the matrix from Exercise 3, click the small blue **`[i]`** button to open the matrix information panel.



In this panel you will see things called **`eigenvalues`** and **`eigenvectors`**.
Use what you observed in the previous exercises to explain what these quantities must represent.
Eigenvalues describe scale. Eigenvectors are directions that are unchanged (or flipped)

- Open the information panel for the other matrices in this exercise, to confirm your answers.

Exercise 6

- At the bottom of the matrix information panel, you can display **`eigenspaces`** for the matrix.

How do the eigenspaces relate to the directions you observed earlier? They show the collection of all eigenvectors for a given eigenvalue