An investigation into Invariance (Part 2)

In this activity you will explore how repeated eigenvalues affect the geometry of transformations. Recall that eigenvectors have their direction preserved under transformation, and they scale by their associated eigenvalue.

- Open Vectorama (www.korovatron.co.uk/vectorama)
- Select **2D** and **ARROW** mode

Exercise 1 • Add a **2D** matrix that enlarges scale factor 2. • Add a **UNIT SQUARE** from the **VECTOR PRESETS**. • Apply the matrix. • Open the matrix information panel [$\boldsymbol{i}$] • Inspect the **eigenvalues** and **eigenvectors** and toggle the **eigenspace** on. • Add several vectors anywhere, in any direction. Apply the matrix. Which vectors are **eigenvectors**? Can you explain your answer?
Exercise 2 • Switch to **3D.** • Add a **3D** matrix that enlarges scale factor 3. • Add a **UNIT CUBE** from the **VECTOR PRESETS**. • Apply the matrix. • Open the matrix information panel [$\boldsymbol{i}$] • Inspect the **eigenvalues** and **eigenvectors** and toggle the **eigenspace** on. • Add several vectors anywhere, in any direction. Apply the matrix. Which vectors are **eigenvectors**? Can you explain your answer?
Exercise 3 • Add the matrix $\begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}$ Inspect the **eigenvalues** and **eigenvectors**. Notice there are only two distinct **eigenvalues**, but three distinct **eigenvectors**. • Add some multiples of the **eigenvector** for the unique **eigenvalue.** • Add several vectors of the form $\alpha \boldsymbol{v}_2 + \beta \boldsymbol{v}_3$, where α, β are scalars and $\boldsymbol{v}_2, \boldsymbol{v}_3$ are the two **eigenvectors** with the same **eigenvalue**. • Toggle the **eigenspace** on and apply the matrix. (You may need to zoom out and rotate for the best view). Describe what you have observed.

An investigation into Invariance (Part 2)

Exercise 4

- Add the matrix $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{pmatrix}$

- Inspect the **eigenvalues** and **eigenvectors** and **eigenspace**.

How is this example different to Exercise 3?

Exercise 5

- Add the matrix $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$

- Inspect the **eigenvalues** and **eigenvectors** and **eigenspace**.

Compare this with Exercise 2. What do you notice?

Exercise 6

Complete the table below

Exercise	2D/3D	Distinct eigenvalues	Independent eigenvector Directions	Shape of eigenspace(s)
1	2D	1	2	All Space R^2
2	3D	1	3	
3	3D			
4	3D			
5	3D			

What conclusions can you make regarding the number of **distinct eigenvalues** and the number of **independent eigenvectors** in relation to **eigenspace**, for any given matrix transformation?